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Mathinks

# MATHLINKS: GRADE 6 STUDENT PACKET 11 RATIOS AND UNIT RATES 

11.1 Ratios

- Define ratio terminology.
- Explore equivalent ratios.
11.2 Unit Rates
- Relate unit rate to ratio
- Represent ratios using symbols, words, tables, and tape diagrams.
- Solve problems using tables and tape diagrams.
- Represent rates using symbols, words, tables, and double number line diagrams.
- Solve problems using rates, tables and double number line diagrams.
11.3 Ratio and Unit Rate Problems
- Solve ratio and unit rate problems using a variety of strategies.
11.4 Skill Builders, Vocabulary, and Review25


## WORD BANK

| Word or Phrase | Definition or Description | Example or Picture |
| :--- | :--- | :--- |
| double number |  |  |
| line diagram |  |  |$\quad$| equivalent ratios |
| :--- |

## RATIOS

## Summary

We will define ratio and explore when ratios are equivalent. We will represent ratios using tables and diagrams, and solve problems involving ratios.

## Goals

- Define ratio terminology.
- Explore equivalent ratios.
- Represent ratios using symbols, words, tables, and tape diagrams.
- Solve problems using tables and tape diagrams.


## Warmup

1. Gretchen was asked to write three fractions that are equivalent to $\frac{3}{7}$. Her work is shown below.

$$
\frac{3}{7}+\sqrt{2}=\frac{2}{9} \quad \frac{3}{7}+\sqrt{3}=\frac{3}{10} \quad \frac{3}{7}+\square \frac{4}{4}=\frac{7}{11}
$$

Explain Gretchen's mistaken thought process.
2. An old television commercial stated that 4 out of 5 dentists surveyed recommended sugarless gum for their patients who chew gum.

Explain, in your own words, what you think this statement means.

## INTRODUCTION TO RATIOS

A ratio is a pair of numbers, not both zero, in a specific order.
The ratio of $a$ to $b$ can be denoted by $a: b$ (read " $a$ to $b$," or "a for every $b$ ").
Example: If there were 3 coins and 2 paperclips in your pocket, then the ratio of the number $(\#)$ of coins to the number (\#) of paperclips is 3 to 2 or $3: 2$. We may also refer to this ratio simply as "the ratio of coins to paperclips."

Write the ratios below for this diagram of circles and arrows.

1. Number of circles to number of arrows

| 2. | Number of circles to total number of shapes | 3 to | or | ___ |
| :---: | :---: | :---: | :---: | :---: |
| 3. | Number of arrows to number of circles | to | or | __ : |
| 4. | Number of arrows to total number of shapes | to | or | _ : |
| 5. | Total number of shapes to number of arrows | to | or | __ : |
| 6. | Number of circles to number of circles | to | or | $\ldots$ _ |

7. The original picture is repeated twice here. There are still 3 circles for every 5 arrows.
a. The new circle to arrow ratio is 6 : $\qquad$

b. Each number in the ratio $3: 5$ can be multiplied by what number to obtain this new ratio? $\qquad$

Two ratios are equivalent if each number in one ratio is a multiple of the corresponding number in the other ratio by the same positive number.

In each arrow diagram below, write the multiplier that can be used to justify that the ratios are equivalent.
8.

9.

10.


## EQUIVALENT RATIOS IN TABLES

When variables in a fixed ratio are represented in tables, pairs of table entries form equivalent ratios.

1. The teacher said that the ratio of the number of fish to the number of frogs in the science lab is 5 to 1 , or 5 fish for every 1 frog.
a. Complete a horizontal table below for possible numbers of animals that could be in the lab.

| \# of fish | 5 | 10 | 30 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| \# of frogs | 1 |  |  | 20 |  |
| Total |  |  |  |  | 600 |

b. The ratios $5: 1$ and $10: 2$ are equivalent ratios because each number in the first pair is multiplied by 2 in the second pair. Using the columns in the table above and the arrow diagrams below, write another ratio that is equivalent to $5: 1$
c. Using columns in the table above, write two ratios that represent the ratio of the number of fish to the total number of animals. Show the multiplier with an arrow diagram.
$\qquad$
to

2. The ratio of the number of 12 -year-olds to the number of 11 -year-olds in the soccer tournament is 1 to 2.
a. Create a vertical table to the right for this situation.
b. Using rows in the table, write two different ratios that are equivalent to $1: 2$. Show they are equivalent with arrow diagrams.

| \# of <br> 11-year <br> olds | \# of <br> 12-year <br> olds | Total |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

c. The ratio of the number of 11 -year olds to the total number of tournament players is
$\qquad$ to $\qquad$ . Choose two ratios for this situation from your table and explain with diagrams and words why they are equivalent.

## EXPLORING RATIOS

1. For an art project in all of her classes, a teacher wants each group to have 4 rulers and 3 glue sticks. What is the ratio of rulers to glue sticks needed for this project?
2. Rodrigo passes out supplies in Period 1. He gives 4 rulers and 3 glue sticks to the first group, 4 rulers and 3 glue sticks to the next group, and continues until he has distributed 24 rulers and a certain number of glue sticks.
a. How many glue sticks did he distribute? $\qquad$
b. Use an arrow diagram to show that a ratio
formed by the number of rulers and glue sticks
distributed is equivalent to the ratio in Problem 1.
3. Pauline passes out supplies to seven groups in Period 2. She says, "I will quickly calculate the numbers of rulers and glue sticks distributed to all the groups first, and then pass them out." Show using ratios how Pauline determined the number of rulers and glue sticks needed.
4. Examine the rows in the table to the right. Are the ratios of feet to eyes equivalent for fish, cows and chicks? Explain.

|  | \# of <br> feet | \# of <br> eyes |
| :---: | :---: | :---: |
| fish | 0 | 2 |
| cows | 20 | 10 |
| chicks | 48 | 48 |

5. Examine the rows in the table to the right. Are the ratios of fingers to ears equivalent in each period? Explain.

|  | \# of <br> fingers | \# of <br> ears |
| :--- | :---: | :---: |
| period 1 | 50 | 10 |
| period 2 | 170 | 34 |
| period 3 | 250 | 50 |
| period 4 | 150 | 30 |

## PRACTICE 1

To the right is a collection of trapezoids and triangles. and $\qquad$ $:$ $\qquad$ $\square \triangle \triangle$ 1. Write two ratios of trapezoids to triangles. $\qquad$ : $\qquad$ $\square$ $\triangle \triangle$
2. Hany circled some shapes from above and said, "I've shown a ratio of 1 trapezoid for every 2 triangles." Circle shapes in the diagram to confirm that Hang is correct.
3. Carter adds two more trapezoids and some triangles to the original collection from above. The ratio of trapezoids to triangles in the collection stays the same.
a. Draw Carter's new collection of shapes to the right.
b. How many more triangles did Carter add? $\qquad$
c. Write the ratio of the trapezoids to triangles for Carter's collection. $\qquad$ : $\qquad$
d. What number could Carter say he used as a multiplier? $\qquad$
e. Explain how you know the ratio in Carter's collection and the original ratio are equivalent.
4. Complete the table below comparing the numbers of trapezoids and triangles in the ratio pictured above. Be sure to include the variables (words) in the left column.

|  | 2 |  |  | 200 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 4 | 12 |  |  |  |
| Total |  |  | 60 |  |  |

5. Itzak continued Hany's work from above and said, "I've shown a ratio of $\frac{1}{2}$ trapezoid for each triangle."
a. Write an explanation or make a drawing to confirm that Itzak is correct.
b. What is the multiplier used to confirm that the original ratio of 2 trapezoids to 4 triangles is equivalent to $\frac{1}{2}$ trapezoid to 1 triangle?

## GRAPE JUICE MIXTURES

Your teacher will give you cards that indicate ways to combine cups of grape concentrate and cups of water to form grape juice mixtures. Discuss the questions below with your partners and record your thinking.

1. Arrange the cards from least "grapey" to most "grapey" and record the letters on the cards here. Note that some of the mixtures may be "equally grapey," or equivalent.

2. Choose a pair of cards and explain how you know which is more grapey.

Critique the reasoning of each student.
3. Jody said, "Mixture A and mixture J will taste the same because they both have the same number of cups of grape."
4. Ed said, "Mixture E and mixture $C$ will taste the same because mixture $E$ has one more cup of water than grape, and mixture $C$ has one more cup of water than grape."
5. Cary said, "Mixture C is more grapey than mixture J because it has more cups of grape."
6. Jon said, "For any mixture, if I double the number of cups of grape and double the number of cups of water, the taste will be the same."

Determine which the following are more, less, or equally grapey when compared to mixture J (2 cups grape : 1 cup water). Explain your answers.
7. The mixture is changed to 2 ounces grape : 1 ounce water.
8. The mixture is changed to 2 gallons grape : 1 gallon water.
9. The mixture is changed to 2 ounces grape : 1 gallon water.

## TAPE DIAGRAMS

A tape diagram is a graphical representation of two variables in which rectangles of equal area represent equal quantities. Tape diagrams are typically used when the quantities have the same units.

The pictures below are both appropriate tape diagrams to represent Card A.
water $\square$

| $G$ | $G$ | $W$ | $W$ | $W$ | $W$ |
| :--- | :--- | :--- | :--- | :--- | :--- |

Draw tape diagrams to represent each indicated juice mixture.

| 1. Card J | 2. Card B |
| :--- | :--- |
| 3. Card F | 4. Card H |

Alex and Andrea were asked how many gallons of grape concentrate and water were needed to make 12 gallons of juice that tastes like mixture A. Their diagrams are below.

| 5. Explain Alex's method: |  |  |  |  |  | 6. Use Alex's method to find the amount of each ingredient needed to make 9 quarts of mixture J . |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| G | G | W | W | W | W |  |
| G | G | W | W | W | W |  |
|  |  |  |  |  |  |  |
| 7. Explain Andrea's method: |  |  |  |  |  | 8. Use Andrea's method to find the amount |
| G | G | W | W | W | W |  |
| 2 | 2 | 2 | 2 | 2 | 2 |  |
|  |  |  |  |  |  |  |

## PRACTICE 2

Use tape diagrams to solve these problems.

1. Milo likes to make fruit soda when he has people over to his house. He has determined that the juice to sparkling water ratio should be $4: 3$. He estimates that he will want 70 cups in all. How much juice and how much sparkling water will he need for the 70 cups?
2. Kendra makes tie-dyed shirts. Her most frequently used colors are orange and green.
a. For the orange dye, she uses red and yellow in a ratio of $3: 2$. How many quarts of red and yellow dye will she need if she wants to make 80 quarts of orange dye?
b. For the green dye, she uses blue and yellow in a ratio of $5: 2$. How many quarts of blue and yellow dye will she need if she wants to make 56 quarts of green dye?

## UNIT RATES

## Summary

We will explore the relationship between ratios and rates. We will define unit rate We will represent ratios and rates with tables and double number line diagrams, and solve problems using rates.

## Goals

- Relate unit rate to ratio.
- Represent rates using symbols, words, tables, and double number line diagrams.
- Solve problems using rates, tables and double number line diagrams.

Joann described the diagram to the right in eight different ways. Circle her statements below that are true.


1. There is a ratio of 3 triangles to 6 squares
2. There is 1 triangle for every 2 squares.
3. There is 1 square for every $\frac{1}{2}$ triangle.
4. If I double the number of triangles, I would also have to double the number of squares to keep the triangle to square ratio the same.
5. If I add 3 more triangles, I also have to add 3 more squares to keep the triangle to square ratio the same.
6. The ratio of squares to total number of shapes is $6: 3$.
7. Triangles represent $\frac{3}{6}$ of the shapes.
8. Triangles represent $\frac{1}{3}$ of the shapes.

## UNIT RATES ASSOCIATED WITH RATIOS

The unit rate associated with the ratio $a: b, b \neq 0$, is the number $\frac{a}{b}$, generally with units of "something per something" attached. The quotient $\frac{a}{b}$ is also referred to as the value of a ratio.

Example: If you drive 40 miles in 2 hours, then your unit rate is $\frac{40}{2}=20$ miles per hour.

1. The ratio of coins to paperclips in the picture to the right is $3: 2$. The unit rate can be written $\frac{3}{2}$ coins per paperclip.

a. The ratio of coins per paperclip can be written $\square$ to 1 .
b. Write the unit rate as a mixed number and as a decimal. $\qquad$
$\qquad$
c. Avi says, "There is no such thing as 1.5 coins." Explain to Avi what this unit rate means using words and pictures.
2. Elyse added one more coin and one more paperclip so that the ratio of coins to paperclips changed. Write the new unit rate of coins per paperclip to prove Elyse's addition changed the initial ratio.


## RATES AND TABLES

1. Here is mileage information for three different vehicles. Use Table I below to complete Table II below.

| TABLE I | Scooter | Bus | Train |
| :---: | :---: | :---: | :---: |
| \# of miles | 100 | 280 | 310 |
| \# of hours | 25 | 7 | 5 |


| TABLE II | Miles to Hours Ratio |  | Unit Rate (in miles per hour) |
| :---: | :---: | :---: | :---: |
| Scooter | miles in__ hours |  |  |
| Bus |  |  |  |
| Train |  |  |  |

2. Explain why the ratios of miles to hours for the three vehicles are not equivalent.
3. The Bead Store sells beads in bags. Use Table III below to complete Table IV below.

| TABLE III | Bag A | Bag B | Bag C | Bag D | Bag E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Price of bag | $\$ 5.00$ | $\$ 2.50$ | $\$ 10.00$ | $\$ 7.50$ | $\$ 12.50$ |
| \# of beads <br> in the bag | 50 | 25 | 100 | 75 | 125 |


| TABLE IV | Bag A | Bag B | Bag C | Bag D | Bag E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Price <br> per bead | $\$ 0.10$ |  |  |  |  |

4. Explain how you know that the ratio of the price per bag to the number of beads in a bag is equivalent for all the bags.

A unit price is a price for one unit of measure.
Example: In problem 3 above, the unit price for beads in each of the bags is $\$ 0.10$ per bead.

## RATIOS AND DOUBLE NUMBER LINES

A double number line diagram is a graphical representation of two variables in which the corresponding values are placed on two parallel number lines for easy comparison. Double number lines are often used when the two quantities have different units.

1. Refer to Table III on page 11 to make a double number line diagram.
price of beads (\$)

2. How are the quantities on the double number line above organized differently from those in Table III on page 11?
3. Alberto jogs 5 yards every 2 seconds.
a. Complete the table below.

| \# of yards | 5 |  |  | 100 |  |  | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# of <br> seconds |  | 4 | 8 |  | 60 | 1 |  |  |

b. Complete each sentence.

Alberto's jogging rate is $\qquad$ yards for one second.

Alberto's unit rate in yards per second is $\qquad$ .

At this rate, Alberto will jog $\qquad$ in one minute.
c. Make a double number line diagram to represent this relationship. You do not need to put every value in the table on the lines.


## RATIOS AND DOUBLE NUMBER LINES (Continued)

4. Ling makes 5 bracelets every 4 hours.
a. Complete the table below.

|  | 5 |  | 25 | 55 | 90 |  | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | 4 | 8 |  |  |  | 1 |  |  |

b. Fill in the blanks:

Ling's rate of making bracelets is $\qquad$ for one hour.

Ling's unit rate in bracelets per hour is $\qquad$ .

At this rate, Ling will make $\qquad$ in 10 hours.
c. Make a double number line diagram to represent this relationship. You do not need to put every value in the table on the lines.

5. Now look at Table I on page 11. Castro says that the collection of traveling rates for the vehicles listed in the table are not equivalent. Therefore, Castro concludes that he cannot record this data on a double number line diagram. Use numbers and words to defend Castro's conclusion.

## POSTER PROBLEMS

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D. I am group member $\qquad$ .
- Each group will start at a numbered poster. Our group start poster is $\qquad$ .
- Each group will have a different color marker. Our group marker is $\qquad$ .

Part 2: Answer the problems on the posters by following your teacher's directions.
Part 3: Return to your seats.

Refer to the poster problems. Discuss and answer each question below.
Each answer has a "twist."

1. Dee Dee can buy 12 markers for $\$ 7.20$. How many markers can she buy for $\$ 1.00$ ?
2. A baseball team plays 8 games in 3 weeks. Devon says, "The games per week unit rate does not make sense." Explain what you think Devon means.
3. A recipe has a ratio of 3 cups of milk to 4 cups of flour. Elisa says combining 3 cups of milk and 4 cups of flour makes a 7 cup mixture. Margit disagrees. Who do you think is correct? Explain.
4. Danisha grew 2 inches in 40 weeks. Explain why this situation could make sense for Danisha, but not for other people.

## PRACTICE

1. The two rate statements below are not the same. Use the concept of unit rates to explain the difference.
a. LaVerne earns $\$ 10$ in 2 hours.
b. Shirley earns $\$ 2$ in 10 hours.

Complete the double number line diagrams below and use the results to answer each question in numbers and words.
2. It takes Joey 5 weeks to read 4 books. At that rate, how many books can he read in 25 weeks?
\# of weeks
\# of books

3. Tommy can buy 2 pounds of bananas for $\$ 0.80$. At that rate, how much would it cost him to buy 16 pounds (lbs) of bananas?

4. Zippy can run 2 miles in 16 minutes. At that rate, how fast can he run 13 miles?
$\qquad$
$\qquad$


## PRACTICE (Continued)

Assume constant rates for each problem.
5. You pay $\$ 60$ for 10 sandwiches.
a. What is the price per sandwich?
b. What is the price for 20 sandwiches at that rate?
6. You read 5 books in 2 weeks.
a. How many books do you read per week?
b. At that rate how many books would you read in 8 weeks?
7. A work day is 8 hours. You earn $\$ 48$ for one work day.
a. What is the hourly pay rate?
b. At that rate how much would you earn in 5 hours?
8. A small fruit basket contains 5 apples, 3 bananas, and 4 oranges.
a. What is the ratio of bananas to fruit in the basket?
b. A larger basket that maintains this ratio of bananas to fruit contains 48 pieces of fruit. How many bananas are in the basket?
9. You eat 2 pounds of potatoes every 8 weeks.
a. How many pounds of potatoes do you eat per week?
b. At that rate how many pounds of potatoes would you eat in 6 weeks?

## RATIO AND UNIT RATE PROBLEMS

## Summary

## Goals

We will solve ratio and rate problems using tables, diagrams, and equations.

- Solve ratio and rate problems using a variety of strategies.

Use tables, diagrams, or unit rates, as appropriate, to solve these problems.
Graham is paid $\$ 90$ for 5 hours of work.

1. At this rate, how much does Graham make for 8 hours of work?
2. At this rate, how long does Graham have to work to make $\$ 63$ ?
3. Graham makes lemonade at work by mixing 3 parts water and 2 parts lemon juice. How much water and lemon juice does he need to make 85 cups of lemonade?

## THE GREEN GROCER

Green Grocer<br>Fruit Special of the Day<br>2 melons for $\$ 3.50$

1. What is the cost per melon?
2. Make a table below to show the cost in dollars of different numbers of melons.

| Cost of melons in <br> dollars (D) | 3.50 |  | 7.00 | 175.00 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| \# of melons (M) | 2 | 6 |  |  | 1 |

3. Make a double number line diagram to show the cost in dollars of different numbers of
melons.
4. Write an equation to compute the cost in dollars $(D)$ of any number of melons $(M)$.
$D=$ $\qquad$
5. Circle numbers that represent the unit rate in problems $1,2,3$, and 4 above.
6. How much will 42 melons cost? Explain.
7. How many melons can be purchased for \$84? Explain.

## THE TOOTHPASTE PROBLEM

Pippy observed that she used 3 tubes of toothpaste in 5 months. At that rate, she wondered how many tubes she would use in a year. Her friends were curious about this too, and they wanted to help her figure this out. Their work is started below.

1. Pippy began the table below. Fill in the missing information and explain her reasoning for this strategy.

| not enough |  |  |  |  |  |  |  | too much |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# of tubes of <br> toothpaste | 3 | 6 | 9 |  |  |  |  |  |  |  |  |
| \# of months | 5 | 10 | 15 | 1 | 2 | 12 |  |  |  |  |  |

2. Tippy made the shorter table below. Fill in the missing information and explain her reasoning based on this table.

3. Zippy likes to do things quickly. Fill in the missing information in the table below and explain his reasoning.

4. How much toothpaste will Tippy use in a year?

## APPLES, APPLES, APPLES

1. You see ads for apples at six different stores. Determine the best buy(s). Show all your work.

2. You want to buy 5 pounds of apples. Explain which store you would choose.
3. Use your understanding of the concept of unit price to interpret each situation.

| Store G: $\$ 0.00$ for 3 apples | Store H: \$3 for 0 apples |
| :--- | :--- |

## BUILDING A DECK

Ben is building a wooden deck that requires 400 feet of lumber. He called several lumber yards to find the best price and wrote down the information below.

| Store A: | The lumber costs $\$ 2.50 / \mathrm{ft}$. | Store B: | Eight feet of lumber costs $\$ 18$. |
| :---: | :---: | :---: | :---: |
| Store C: | The total for all 400 feet is $\$ 910$. | Store D: | The total for the first 200 feet is $\$ 500$. After that, the cost is \$1.90/ft. |
| Store E: | The first 300 feet cost $\$ 2.80 / \mathrm{ft}$. Aft | that, the r | rate is $\$ 0.75 / \mathrm{ft}$. |

Show your work above and explain your reasoning below.

1. Which store provides Ben with the best buy to purchase 400 feet of lumber?

How much will this cost Ben?
2. If you worked at a store and wanted to make Ben a better deal, what unit price (per foot) could you offer him for your lumber?

## THE GRAIN GROCER

## The Grain Grocer sells rice in bulk.

The special of the day is to the right.
Antonio said, "The ratio of the number of dollars to the number of pounds is $4: 5$. That's $\$ 0.80$ per

Grain Grocer
Special of the Day pound."

Bianca said, "The sign means that the ratio of the number of pounds to the number of dollars is $5: 4$. That's 1.25 pounds per dollar."

1. Are Antonio and Bianca both correct? Explain.
2. Carlos needs two pounds of rice to make a casserole. Explain to Carlos how much money he will need.
3. Dora has $\$ 10$ and wants to stock up on rice. Explain to Dora how many pounds of rice she can buy.
4. Do you prefer to answer problems 2 and 3 using Antonio's rate of $\$ 0.80$ per pound, using Bianca's rate of 1.25 pounds per dollar, or using another strategy? Explain.

## THE ASSEMBLY

The school auditorium has 330 seats. When the $6^{\text {th }}$ grade students went to assembly period 1 , seats were filled at a ratio of 9 occupied to 2 unoccupied. When the $7^{\text {th }}$ grade students went to assembly period 2 , seats were filled at a ratio of 5 occupied to 1 unoccupied.

1. How many $6^{\text {th }}$ grade students went to the assembly?
2. How many $7^{\text {th }}$ grade students went to the assembly?
3. If $3008^{\text {th }}$ grade students went to assembly period 3 , what was the ratio of occupied to unoccupied seats?

## THE PAINT MISTAKE

Jimbo's daughter says she wants to paint her room pink. He thinks the color can be made by using 2 scoops of white paint for every 3 scoops of red paint. When Jimbo makes a sample using exactly 5 scoops, his daughter says, "No, you were supposed to use 3 scoops of white paint for every 2 scoops of red paint."

Jimbo does not want to waste any paint. Without throwing out the 5 scoop mixture he already made, describe how Jimbo can correct the mistake when making a larger batch of paint the way his daughter wants it.

## SKILL BUILDERS, VOCABULARY, AND REVIEW <br> SKILL BUILDER 1

Compute the following sums and differences.

| 1. $\frac{1}{4}+\frac{1}{3}$ | 2. $\frac{3}{5}-\frac{1}{10}$ | 3. $\frac{3}{4}+\frac{5}{6}$ | $4.4 \frac{3}{4}-4 \frac{1}{2}$ |
| :--- | :--- | :--- | :--- |

5. Estimate the location of each number on the number line below.
$\frac{7}{7} \quad 1 \frac{2}{3}$
$\frac{6}{5}$
$\frac{2}{3}$
$\frac{4}{3}$
$1 \frac{1}{5}$
$\frac{2}{3}$
$\frac{1}{12}$
6. Which of the numbers from problem 5 above are equal? $\qquad$
7. Javier incorrectly thinks that 0.3 and $\frac{1}{3}$ are equal. Explain to Javier why they are not.

Rewrite each number below in its equivalent representations.

|  | Fraction or mixed number | Decimal | Percent |
| :---: | :---: | :---: | :---: |
| 8. | $\frac{1}{8}$ |  |  |
| 9. |  | 0.02 |  |
| 10. |  |  | $18.9 \%$ |
| 11. |  |  |  |

## SKILL BUILDER 2

The data in the table below represents the number of division problems individual students answered correctly in two minutes.

| 38 | 42 | 46 | 46 | 38 | 43 | 39 | 74 | 36 | 46 | 35 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 66 | 45 | 49 | 45 | 46 | 46 | 46 | 73 | 52 | 50 | 42 | 45 |

1. Arrange the data from above in numerical order in the table below.

2. Make a dot plot for the data using the number line below.

3. Find the five-number summary for the data.

4. Construct a box plot below for this data.

5. Construct a histogram below for this data.

6. Does the data skew left or right? $\qquad$ . What does this say about the number of division problems the class can do in 2 minutes?

## SKILL BUILDER 3

Simplify each expression first if possible. Then complete the table below based on the simplified expressions.

| Expression | Number <br> of terms | Constant <br> term(s) | Term(s) with <br> Variables | Coefficient of <br> the variable(s) |
| :---: | :---: | :---: | :---: | :---: |
| 1. $6+5 m-2$ |  |  |  |  |
| 2. $w+w+w+w+w+w$ |  |  |  |  |

Evaluate each expression for $w=5, x=2$ and $y=3$.
3. $w+(x+y)^{3}$
4. $(x y-w)^{7}$

Use exactly one 3 , one 5 , and one 2 , along with any mathematical operations and symbols, to create expressions that meet the following conditions.
5. The target number is greater than 100
6. The target number is even and greater than 10.
7. The target number is a prime number.

Solve using any strategy. Write MM if mental math is used. Otherwise show your work.

| 8. | $x-3.5=7.05$ | 9. | $y+1.7=2.1$ |
| :--- | :--- | :--- | :--- |

Compute.

| 11. | $1092 \div 12$ | 12. |
| :--- | :--- | :--- |
|  |  | $52.38 \div 5.4$ |

## SKILL BUILDER 4

1. Explain in your own words what this statement means: the ratio of the number of blue marbles to the number of red marbles in a bag is 2 to 3 .

| Here are some arrows and stars: |
| :--- |
| Write each ratio. |
| 2. The number of arrows to the number of stars |
| 3. The number of stars to the number of arrows |
| 4. The number of stars to the total number of <br> shapes |
| 5. The total number of shapes to the number of <br> arrows |

6. Charlie adds six more stars to the collection of shapes above. How may arrows would he need to add to keep the ratio of arrows to stars the same? Explain.
7. The ratio of women to men in a coed volleyball tournament is 2 to 3 .
a. Create a table below for this situation.

|  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |
| Total |  |  |  |  |  |

b. Using the columns in the table above, write two different ratios and explain how you know these ratios involving numbers of people in the tournament are equivalent. Use an arrow diagram if desired.

## SKILL BUILDER 5

Franco likes to make a lemon tea that is in a ratio of 4 cups of lemonade to 5 cups of iced tea. He is hosting a party for 15 people and estimates that each person will drink 3 cups of lemon tea.

1. How much lemon tea should he make so every guest has 3 cups?
2. Construct a tape diagram to figure out how many cups of lemonade and iced tea Franco should use.
3. How many cups of lemonade should Franco put in the lemon tea?
4. How many cups of iced tea should Franco put in the lemon tea?

To help him make different amounts of lemon tea, Franco started the table below.
5. Complete the table.

| lemonade <br> (\# of cups) | 4 |  |  | 16 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| iced tea <br> (\# of cups) | 5 |  | 15 |  |  |
| Total (\# of cups) |  | 18 |  |  | 144 |

6. Which of the recipes below could be used to make Franco's lemon tea?

| a. 2 cups of lemonade <br> 2.5 cups of iced tea | b. 5 cups of lemonade <br> 4 cups of iced tea |
| :--- | :--- |
| c. 20 cups of lemonade |  |
| 25 cups of iced tea |  |$\quad$| d. $1 \frac{1}{4}$ cups of iced tea |
| :--- |
| 1 cup of lemonade |

## SKILL BUILDER 6

1. Give an example of a rate.
2. You work 10 hours and make $\$ 118$. Complete the table below. Then circle the column in the table that could be used to represent your hourly pay rate (unit rate).

| pay (\$) | 118 | 236 |  | 1180 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# of hours | 10 |  | 2 |  | 100 | 1 | 5 | 25 |

An 8 pound bag of dog kibble contains about 26 cups of kibble. Barry's dog Cocoa eats
$2 \frac{1}{2}$ cups of kibble per day. Use this information to complete problems 3-6.
3. Make double number lines below to represent these ratios.

cups of kibble
\# of days

4. How many cups of kibble does Cocoa eat in two weeks?
5. The table to the right shows the cost of kibble in various-size bags.

Do the costs per pound for the bags represent equivalent ratios? $\qquad$ Explain.
6. If Barry wants to buy Kibble for Cocoa that will last for two weeks, what should he purchase?

| Bag size <br> (in pounds) | Cost <br> (\$) |
| :---: | :---: |
| 4 | 6.00 |
| 8 | 10.00 |
| 20 | 20.00 |

## SKILL BUILDER 7

1. A shade of pink paint requires 5 parts of red paint and 2 parts of white paint. Use a table or diagram for the following.
a. How much red paint should be used if you have 1 gallon of white paint?
b. How much white paint should you use if you have 25 gallons of red paint?
2. Miko wants to make a whiter shade of pink paint. Write two ratios (red paint to white paint) that would make a whiter shade of pink compared to the mixture in problem 1.

Explain your reasoning.
3. Stephen wants to make a redder (darker) shade of pink paint. Write two ratios (red paint to white paint) that would make a redder (darker) shade of pink compared to the mixture in problem 1.

Explain your reasoning.

DeSean and Danny both wash cars on weekends to earn extra money. DeSean washes at a rate of 5 cars per every 2 hours. Danny washes 7 cars per every 3 hours.
4. Who washes cars at a faster rate?
5. DeSean hurts his arm, so for the next several weekends he is washing 5 cars per every 2.5 hours. Use a table or diagram and explain how his injury changes the answer above.

## SKILL BUILDER 8

1. Hugo is buying cupcakes for a birthday party. At Cupcake Land, he can buy 6 cupcakes for \$5. At Cake-in-a-Cup, he can buy 9 similar cupcakes for $\$ 8$. Which store offers Hugo a better buy? Explain.
2. Harvey says that The Sweet Shop, offering 12 cupcakes for $\$ 10$, is a better deal than the choices in problem 1 because he can get the most cupcakes. Critique Harvey's reasoning.
3. Minuet gets $\$ 30$ in allowance every two weeks. She is saving all of her allowance to buy a prom dress that costs $\$ 210$. How many weeks will it take her to save up enough money for the dress? Use a table, tape diagram, or double number line to show your reasoning.
4. Lexie is making an orange punch drink for a party. She puts 4 pints of orange drink and 2 pints of water into a punch bowl. Her friend Olivia says, "Oh no, you were supposed to make a mixture that is 3 parts orange drink and 4 parts water." How can Lexie add to the current punch bowl mixture and correct the mistake so that the orange drink mixture is done the way Olivia suggests.

## FOCUS ON VOCABULARY



Across
Down
15 is a $\qquad$ used to show that 2:3 2 the value of a ratio: $\qquad$ rate and 15:15 are equivalent.

6 a pair of numbers in a specific order
3 A(n) $\qquad$ diagram shows equivalent ratios when units are the same.

8 Equivalent ratios are examples of this kind of relationship
$4 \quad \$ 3$ per 1 melon is an example of a unit
$\qquad$ -.
$5 D=1.75 M$ and $3 x+6=9$ are examples of $\qquad$ .
$7 \quad \mathrm{~A}(\mathrm{n})$ $\qquad$ number line shows equivalent ratios when units are different.

## SELECTED RESPONSE

Show your work on a separate sheet of paper and select the best answers).

1. Mrs. Robert's math class has 13 boys and 11 girls. Which of the following shows the relationship of the number of girls to the total number of students in the class? Choose ALL that apply.
A. $13: 11$
B. $11: 13$
C. 11:24
D. $24: 13$

For problems 2 and 3 , use the collection of shapes to the right:

2. Which of the following represent the ratio of triangles to circles? Choose ALL that apply.
A.
3 to 2
B. 2 to 3
C. 3 to 5
D. 1.5 to 1
3. Carlos added 4 more circles and some triangles to the collection of shapes above. The ratio of triangles to circles stayed the same. How many triangles did Carlos draw?
A. 3
B. 4
C. 5
D. 6

For problems 4-6: Jacqueline worked 40 hours and made $\$ 480$.
4. What was Jacqueline's hourly pay rate? Choose ALL that apply.
A. $\frac{480}{40}$ dollars per hr
B. $\$ \frac{480}{40}$ per hr
C. $\$ 12$ per hr
D. $\$ \frac{1}{12}$ per hr
5. At the rate stated in problem 4, how much would Jacqueline expect to make in 160 hours?
A. $\quad \$ 120$
B. $\$ 192$
C. $\$ 960$
D. $\$ 1,920$
6. At the rate stated in problem 4 , how many hours would Jacqueline work to make $\$ 288$ ?
A. 12
B. 20
C. 24
D. 240
7. Jonette knows that the ratio of blue marbles to red marbles in a bag is $3: 4$. How many blue and red marbles could be in the bag? Choose all that apply.
A. 5 blue, 6 red
B. 6 blue, 8 red
C. 27 red, 36 blue
D. 27 blue, 36 red

## KNOWLEDGE CHECK

Show your work on a separate sheet of paper and write your answers on this page.

### 11.1 Ratios

Here are some stars and arrows:


1. What is the ratio of the number of stars to the total number of shapes?
2. Use words to describe what the ratio $5: 9$ means in this context.
3. If George added 18 shapes to the collection but the ratio of stars to arrows remains the same, how many stars did he add?

### 11.2 Unit Rates

A horse eats 132 bales of hay in 12 months. Answer the following questions. Use a table or diagram to show your reasoning.
4. How many bales of hay does the horse eat per month?
5. At that rate, how many bales of hay would the horse eat in 8 months?
6. How much hay would 3 horses eat in 12 months?
7. How long would 198 bales last if it were used to feed one horse?

### 11.3 Ratio and Unit Rate Problems

Maria earns the same amount each day for her part time job. After working 8 days she made \$1,200.
8. Use a double number line diagram to find out how much she made in 6 days.

9. How much will she make in 40 days? Show your work.

## HOME-SCHOOL CONNECTION

Here are some problems to review with your young mathematician.
About 1 out of 10 people are left-handed.

1. Make a tape diagram to represent this situation.
2. In a group of 70 people, about how many people would you expect to be left-handed? Explain how to solve this problem using the tape diagram.

Raymond walked in a fundraiser for cancer research. He walked 18 miles in 6 hours and raised $\$ 1,440$.
3. How many dollars per hour walking did he raise?
4. Create a double number line diagram to show the relationship between the number of miles walked and the number of hours walked.
5. Use your double number line to determine how long it took for him to walk 12 miles.
$\qquad$

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## COMMON CORE STATE STANDARDS - MATHEMATICS

| STANDARDS FOR MATHEMATICAL CONTENT |  |
| :---: | :---: |
| 6.RP.A 6.RP. 1 | Understand ratio concepts and use ratio reasoning to solve problems. <br> Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate $A$ received, candidate $C$ received nearly three votes." |
| 6.RP. 2 | Understand the concept of a unit rate $a / b$ associated with a ratio $a: b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of $\$ 5$ per hamburger." |
| 6.RP.3a | Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. |
| 6.RP.3b | Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? |
| STANDARDS FOR MATHEMATICAL PRACTICE |  |
| MP1 | Make sense of problems and persevere in solving them. |
| MP3 | Construct viable arguments and critique the reasoning of others. |
| MP4 | Model with mathematics. |
| MP7 | Look for and make use of structure. |



